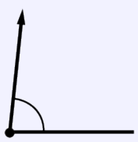
Grade 6 Math Circles
March 28/29/30, 2023
All About Angles - Solutions

Exercise Solutions

Exercise 1

Classify the following angles:

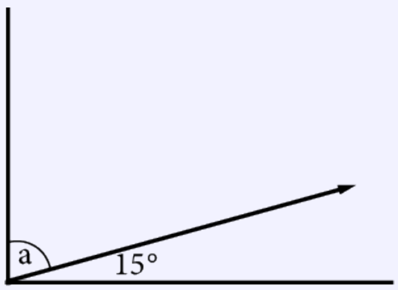
1. 
2. 
3. 
4. 

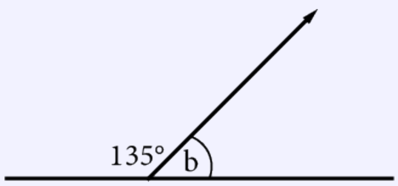
Exercise 1 Solution

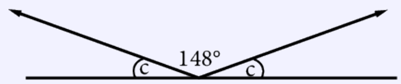
1. Acute angle
2. Reflex angle
3. Reflex angle
4. Straight angle

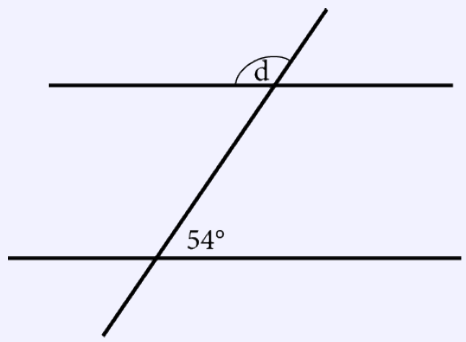
Exercise 2

Determine the measure of the unknown angles, then state which property you used:

1. 

2. 

3. 

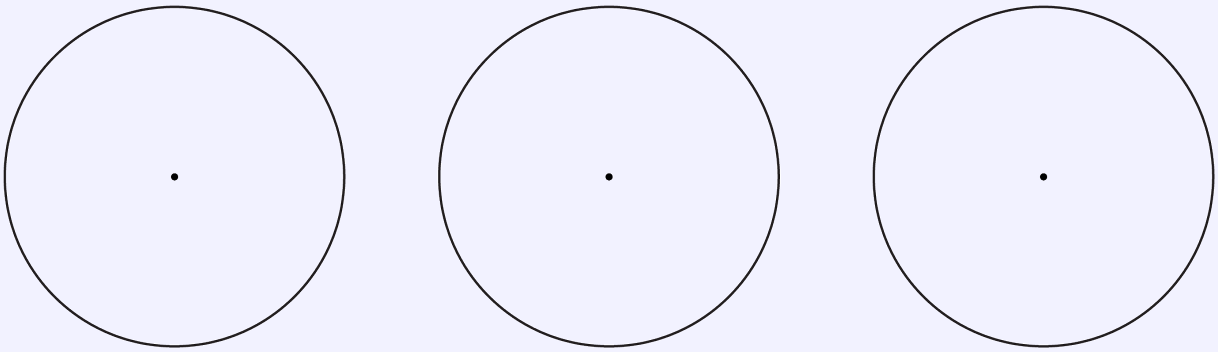
4. 

Exercise 2 Solution

1. a = 75° due to the complementary angle property
2. b = 45° due to the supplementary angle property
3. a = 16° due to the straight angle and supplementary angle properties
4. a = 126° due to corresponding and supplementary angle properties

Exercise 3

Pick any points A and B on the circumference of the circle, and draw an inscribed angle and a central angle from them on the same circle. Do this three times, with different points A and B. Write down the measure of your angles. What do you notice?

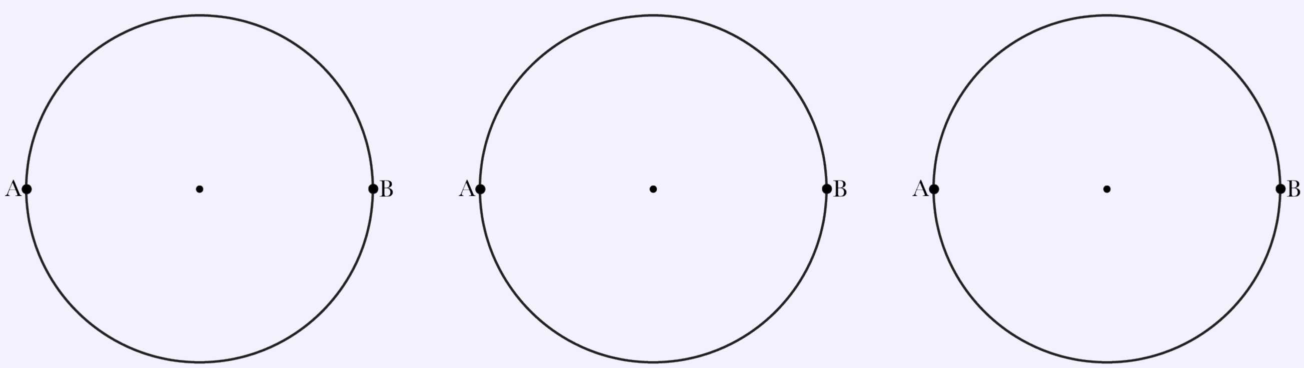


Exercise 3 Solution

Solutions may vary. You should notice that for all three of your angle sets, the central angle is exactly double your inscribed angle.

Exercise 4

Draw three inscribed angles from the diameter (all with a different vertex). Write down the measure of your angles. What do you notice?

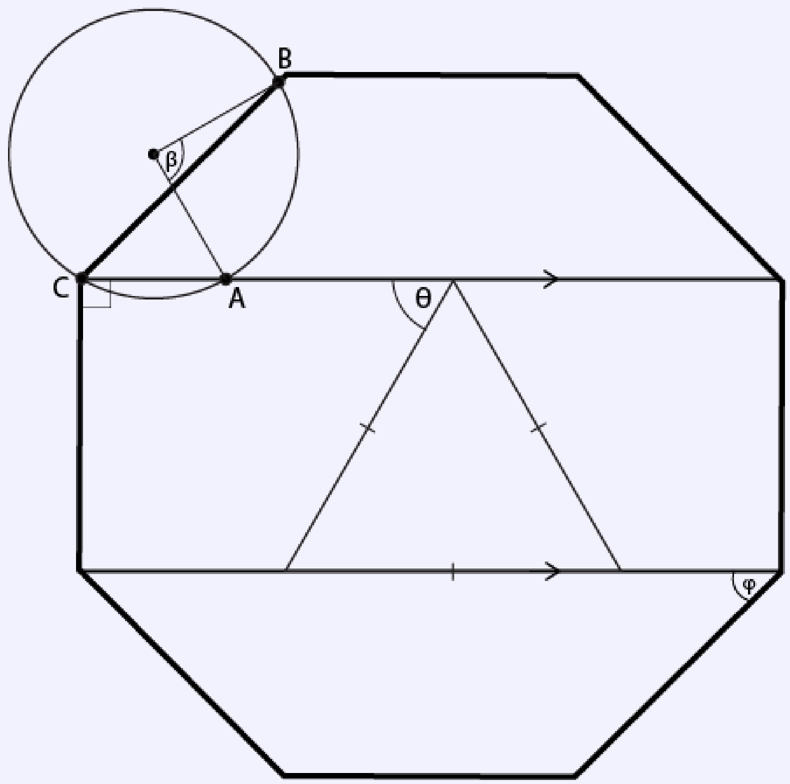


Exercise 4 Solution

Solutions may vary. You should notice that all three inscribed angles from the diameter have a measure of exactly $90°$.

Exercise 5

Determine the measure of **all** the indicated angles (θ, β, φ) inside the regular octagon given below:



Exercise 5 Solution

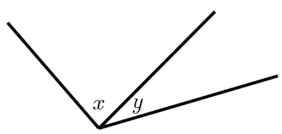
In the middle of the octagon, there is an equilateral triangle, whose interior angles are all $60°$. Using the "Z-pattern", we can say that $\theta = 60°$.

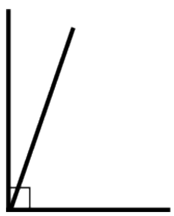
Each interior angle inside a regular octagon is $135°$. Further, note that φ is adjacent to a right angle. Hence: $\varphi = 135° - 90° = 45°$.

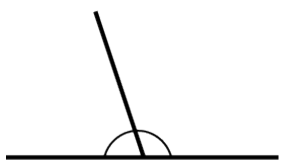
Finally, notice that β is a central angle with a related inscribed angle at point C. This inscribed angle is $45°$ using the same reasoning as for φ, and thus $\beta = 2 \times 45° = 90°$.

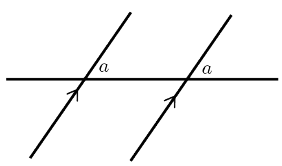
Problem Set Solutions

1. Classify the following angles:

(a) 

(b) 

(c) 

(d) 

Solution:

(a) Adjacent angles

(b) Complemetary angles, both of which are acute.

(c) Supplementary angles. One is acute, one is obtuse.

(d) Corresponding acute angles.

2. What is the

   (a) Supplementary angle to $32°$?

   (b) Complementary angle to $32°$?

   (c) Supplementary angle to $79°$?

   (d) Supplementary angle to $180°$?

Solution:

(a) $148°$

(b) $58°$

(c) $101°$

(d) $0°$

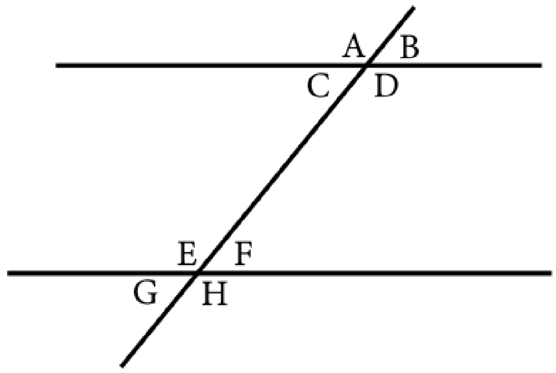
3. Two angles are complementary to each other. One is $12°$ larger than the other. What are the angles?

Solution: Let x represent the smaller angle. Then the larger angle can be expressed as $x + 12$. Since they are complementary, the two angles must sum to $90°$, so we solve the following equation:

$$\begin{aligned} x + x + 12 &= 90° \\ 2x + 12 &= 90° \\ 2x &= 78° \\ x &= 39° \end{aligned}$$

Since the smaller angle is $39°$, the larger angle must be $39° + 12° = 51°$.

4. Consider the following diagram:



(a) List all the pairs of supplementary angles

(b) How many angles need to be known if you want to determine *all* the angles in the diagram?

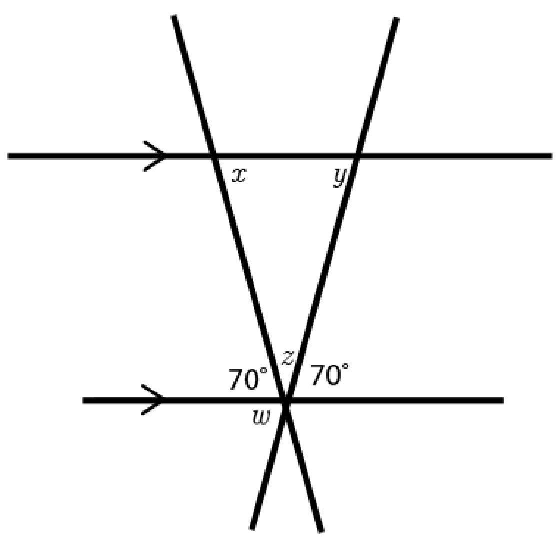
Solution:

(a) A,B ; C,D ; A,C ; B,D ; E,F ; E,G ; F,H ; G,H

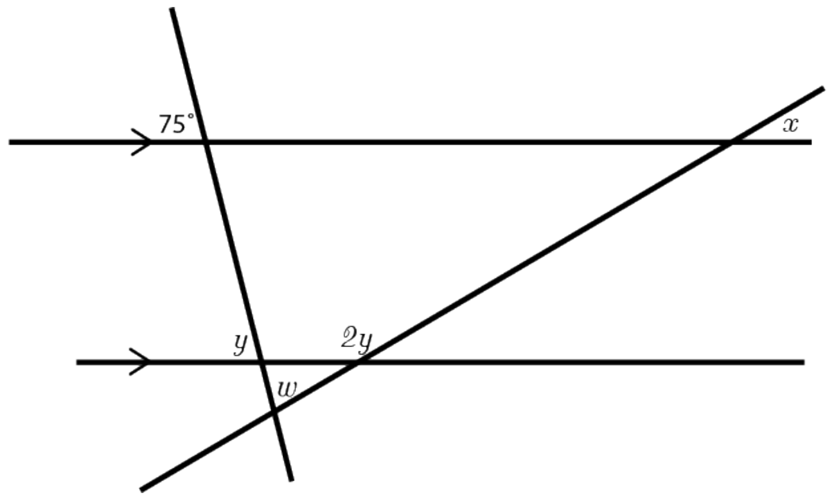
(b) Only one angle needs to be known! Suppose you were given the measure of A. By supplementary, opposite, corresponding, and alternate angle properties, you can determine the measures of angles B, C, D, E, and H. Angles G and F can then be determined by using angle properties with angles E and H.

5. Determine the missing angles using angle properties:

a)



b)



Solution:

(a) $x = 70°$, $y = 70°$ due to alternate interior angle properties.

$w = 70°$ due to the opposite angle property.

Use interior angles of a triangle to solve for z: $z = 180° - 70° - 70° = 40°$

(b) $y = 75°$ due to the corresponding angle property, and so $2y = 2 \times 75° = 150°$.

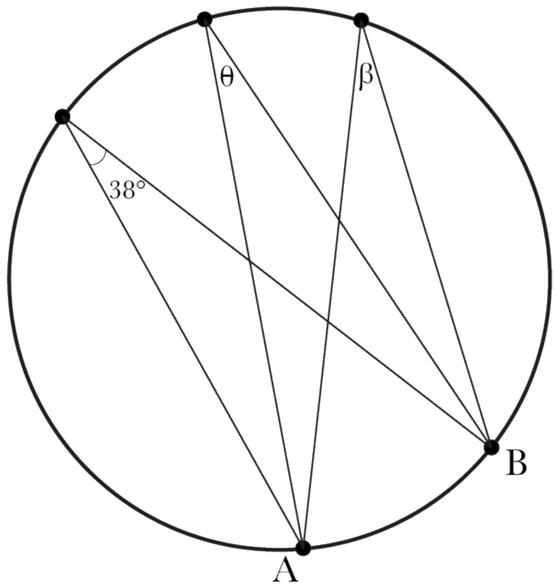
To solve for x, notice that the supplementary angle to x is a corresponding angle to $2y$, thus $x = 180° - 150° = 30°$.

To solve for w, the other two angles within the triangle containing w must be known. Notice that one of these angles is an opposite angle to y and so it must be $75°$, and the other missing angle is an alternate exterior angle to x and so must be $30°$.

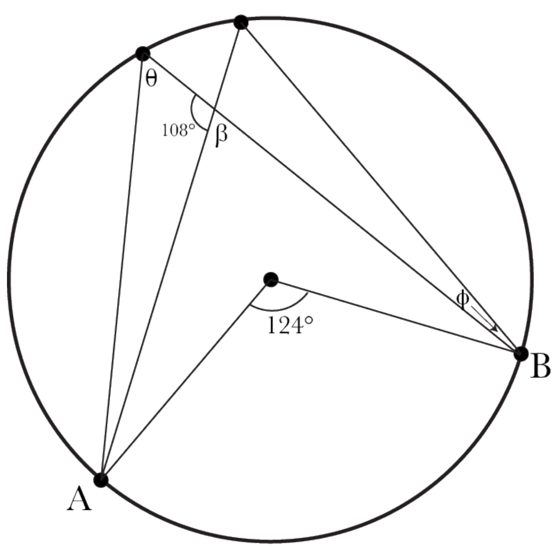
$w = 180° - 75° - 30° = 75°$.

6. Determine the missing angles using angle properties:

a)



b)



Solution:

(a) $\theta = 38°$, $\beta = 38°$ (Inscribed angle relationship)

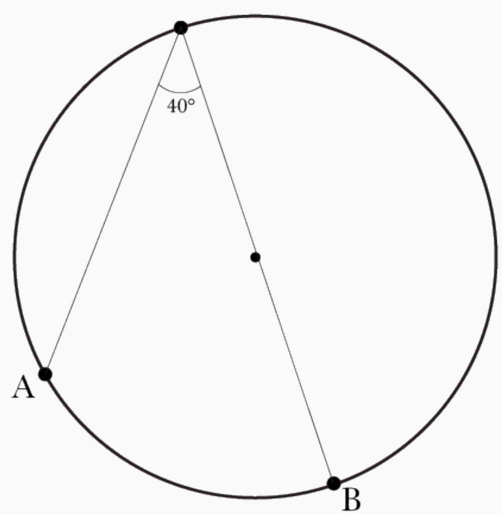
(b)

$$\begin{aligned}\theta &= \frac{124°}{2} && \text{(Inscribed angle and central angle relationship)}\\ &= 62°\\ \beta &= \frac{360° - (108° \times 2)}{2} && \text{(Opposite angle and complete rotation property)}\\ &= 72°\\ \Phi &= 180° - 108° - 62° && \text{(Interior Angles of a triangle)}\\ &= 10°\end{aligned}$$

7. Draw the following angles inside of a circle:
   (a) An inscribed angle with a related central angle of 80°.
   (b) A central angle with a related inscribed angle of 63°.
   (c) An inscribed angle from the diameter whose vertex is equidistant from points A and B.

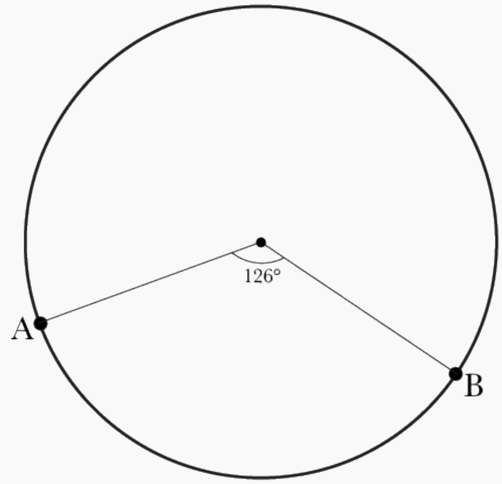
Solution:

Solutions may vary.

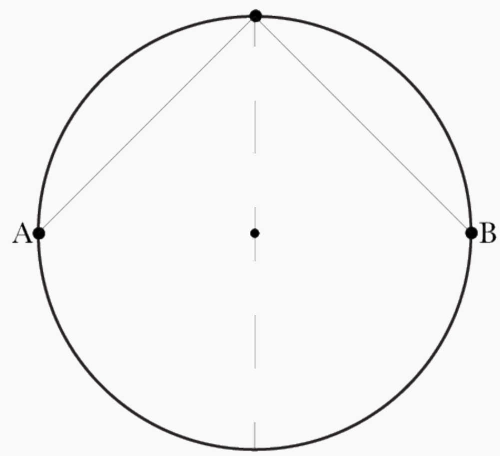


(a)

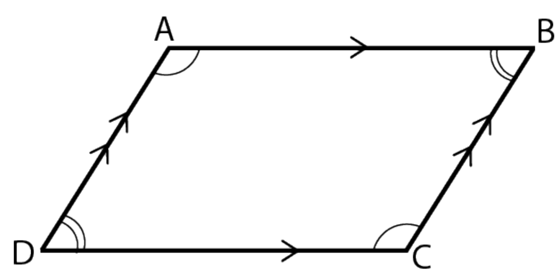
(b)



(c)



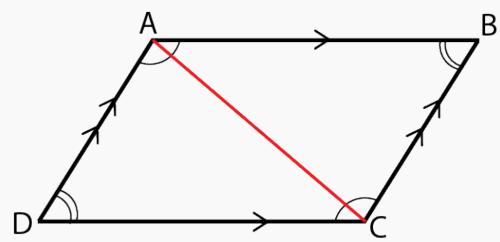
8. (a) Consider the parallelogram below. Can you draw a line to break up the shape into two triangles?



(b) Use angle properties and interior angle rules to prove that the sum of the interior angles in a parallelogram is 360°.

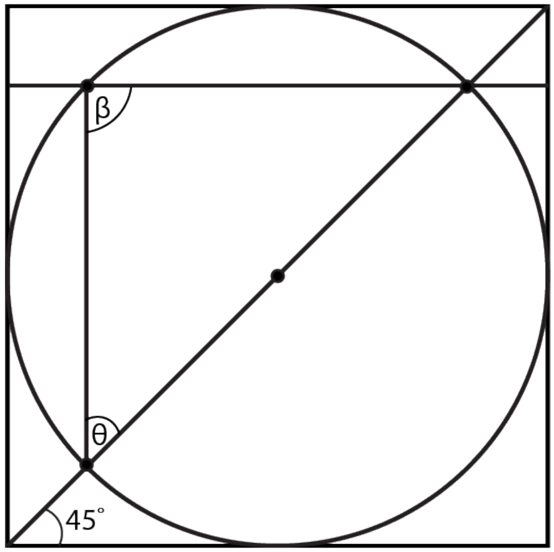
Solution:

(a) Yes, it is possible:



(b) First, consider the fact that the line we drew breaks up the parallelogram into two triangles. Further, angles A and C have been divided into two adjacent angles that add up to A and C. Thus, the interior angles of the two triangles add up to the interior angles of the parallelogram. Since the interior angles of a triangle sum up to 180°, the sum of the interior angles of a parallelogram is: $180^\circ + 180^\circ = 360^\circ$.

9. Determine the missing angles using angle and shape properties:



Solution:

$\beta = 90^\circ$ (Inscribed angle from the diameter rule)

$\theta = 180^\circ - 45^\circ - 90^\circ$ (Alternate angle property + sum of the interior angles of a triangle)